Effect of Longitudinal Stringers on Sound Transmission into a Thin Cylindrical Shell

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In the context of the transmission of airborne noise into an aircraft fuselage, a mathematical model is presented for the transmission of airborne noise into a stiffened cylindrical shell. The stiffeners are longitudinal stringers and are modeled as discrete structural elements. The numerical cases examined were typical of a narrow-bodied jet transport fuselage. The stringers appeared to raise the cylinder transmission loss in the mass-controlled region, although they produced dips at the stringer resonances. The ring-frequency dip in transmission loss, which is characteristic of monocoque shells, was found to still be present. There appeared to be a small increase in transmission loss as the number of stringers was increased. The effect of high damping was also investigated.

I. Introduction

THE work reported herein is an extension of an earlier study[1] of the sound transmission through a monocoque shell structure. In the context of airborne-noise transmission through an aircraft fuselage, the specific problem studied was that of an incident oblique plane wave impinging upon a flexible cylindrical shell. The work in Ref. 1 was an extension of earlier work of Smith[2] and treated the "fuselage" as a monocoque shell.

An attempt was made[3] to improve the model by including the effects of ring frames and stringers by means of a "smeared-stiffener" theory formulated by Rosen and Singer.[4] However, the effort met with only limited success, although Rosen and Singer demonstrated in Ref. 4 that good results were obtained in shell vibration studies. Apparently, the smeared-stiffener approach, while valid at low frequencies, breaks down at the higher frequencies of interest in sound transmission work. To correct this, the present work replaces the smeared-stiffener theory by one which treats the stringers as discrete elements, in the manner of Rinehart and Wang,[5,6] and studies the effects of the longitudinal stiffeners on the transmission of airborne noise.

II. Mathematical Model

The model derived in Ref. 1 is summarized below, where the cylinder transmission loss (TL), defined as the ratio between the incident acoustic power and the power absorbed by the shell contents, is given by

$$\mathrm{TL} = -10\log_{10} Q \tag{1}$$

where

$$Q = \sum_{m=0}^{\infty}\left(\frac{2\epsilon_m}{x_{1r}}\right)\frac{r_m^c r_m^s}{(r_m + r_m^s)^2 + (\chi_m + \chi_m^s)^2} \tag{2}$$

$$r_m = \mathrm{Re}(z_m),\ \ \chi_m = \mathrm{Im}(z_m) \tag{3}$$

$$r_m^s = (2/\pi x_{1r})/(J_m'^2 + Y_m'^2) \tag{4}$$

$$\chi_m^s = -(J_m J_m' + Y_m Y_m')/J_m'^2 + Y_m'^2) \tag{5}$$

$$x_{1r} = k_{1r} a \tag{6}$$

$$k_{1r} = (\omega/c_1)\sin\theta/(1 + M_1\cos\theta) \tag{7}$$

$$z_m = (Z_m/\rho_1 c_1)\sin\theta(1 + M_1\cos\theta) \tag{8}$$

$$\epsilon_m = \begin{cases} 1 & (m=0) \\ 2 & (m \geq 1) \end{cases} \tag{9}$$

In Eqs. (1-9), a is the shell radius; θ is the incidence angle of the incident plane wave (see Fig. 1); $J_m = J_m(x_{1r})$ and $Y_m = Y_m(x_{1r})$ are Bessel functions of the first and second kinds of order m; $M_1 = V/c_1$ = external flow Mach number; ρ_1 and c_1 are the fluid density and sound speed in the external medium; and ω is the frequency of the incident sound wave. The impedance of the shell and its contents, Z_m, is given by

$$Z_m = Z_m^{sh} + Z_m^c \tag{10}$$

where Z_m^{sh} is the modal impedance of the shell, and Z_m^c is that of its contents. If it is assumed that the shell interior is totally absorptive, then only inward-traveling waves exist (in direct analogy with the classical problem of sound transmission through a flat panel), and Z_m^c can be represented by

$$Z_m^c = j\rho_2 c_2 H_m^{(1)}(x_{2r})/H_m^{(1)\prime}(x_{2r}) \tag{11}$$

where $x_{2r} = k_{2r} a$, and

$$k_{2r} = (\omega/c_2)\,[1 - (c_2/c_1)^2\cos^2\theta/(1 + M_1\cos\theta)^2]^{1/2} \tag{12}$$

In Eq. (12), ρ_2 and c_2 are the fluid density and sound speed in the interior medium, and $H_m^{(1)}$ is the Hankel function[7] $H_m^{(1)} = J_m + jY_m$ of order m.

It remains to determine the shell impedance Z_m^{sh}. Appropriate equations for a discretely stiffened shell are derived in the Appendix, and Z_m^{sh} is shown to be given by

$$Z_m^{sh} = j\, m^{sh}\omega \frac{(\omega_R/\omega)^2\Delta}{(1-\nu^2)\,[A_{12}A_{21} - A_{11}A_{22}]} \tag{13}$$

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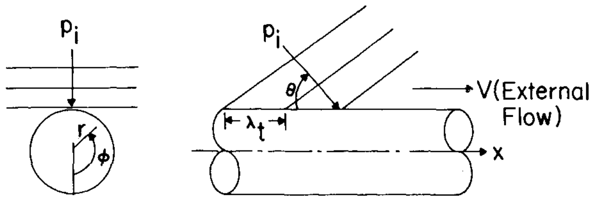

Fig. 1 Geometry of problem studied.

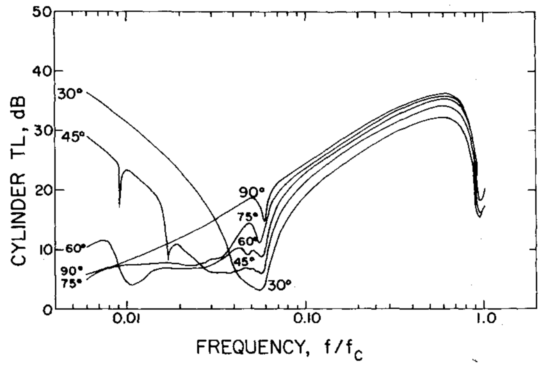

Fig. 2 Cylinder TL for various incidence angles, monocoque shell.

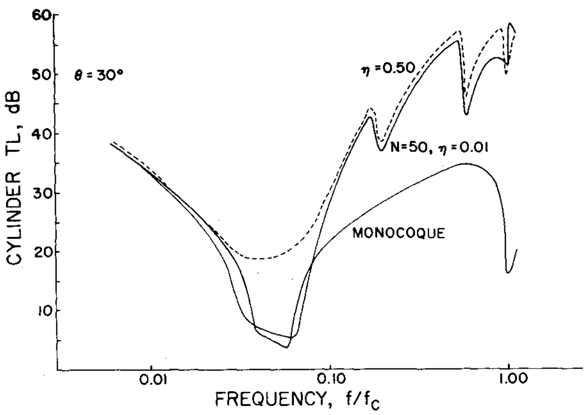

Fig. 3 Cylinder TL for stiffened shell ($\theta = 30$ deg, $N = 50$).

where m^{sh} is the mass per unit area of the shell and ω_R is the ring frequency for a monocoque shell, given by

$$\omega_R = a^{-1} (E/\rho^{sh})^{1/2} \tag{14}$$

where E is Young's modulus and ρ^{sh} is the shell material density. The factors A_{ij} and Δ are defined in the Appendix. The goal of the present investigation can now be stated as a study of the frequency spectrum of TL in Eq. (1) and of how TL for a stiffened shell compares with that for a monocoque shell.

III. Numerical Results

Numerical results have been generated for various incidence angles θ for an aluminum cylinder with radius $a = 1.83$ m (6 ft) and wall thickness $h = 0.159$ cm (1/16 in.). Both internal and external fluids were assumed to be the same: $\eta = 0.01$ unless otherwise noted, and $M_l = 0$. The longitudinal stiffener employed in the calculations is made of aluminum and consists of an inverted hat section 2.54 cm (1 in.) wide, 3.18 cm (1.25 in.) high, with feet 1.78 cm (0.70 in.) wide. The stiffener is 0.18 cm (0.071 in.) thick. These numbers are typical of those used in narrow-bodied passenger jets. Unless otherwise specified below, the preceding values are used for stringer dimensions.

In the following discussion, the basic reference is the monocoque shell. For this reason, Fig. 2 (reproduced from

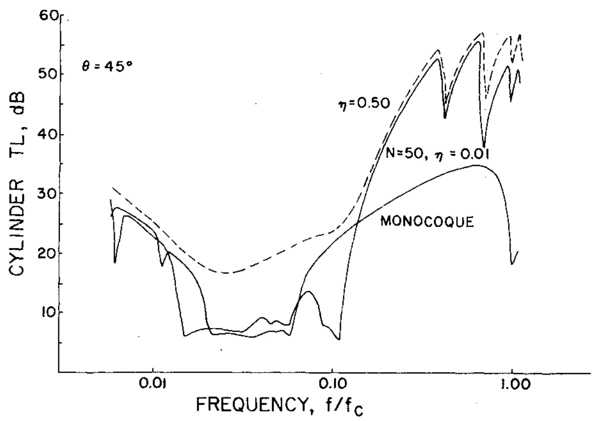

Fig. 4 Cylinder TL for stiffened shell ($\theta = 45$ deg, $N = 50$).

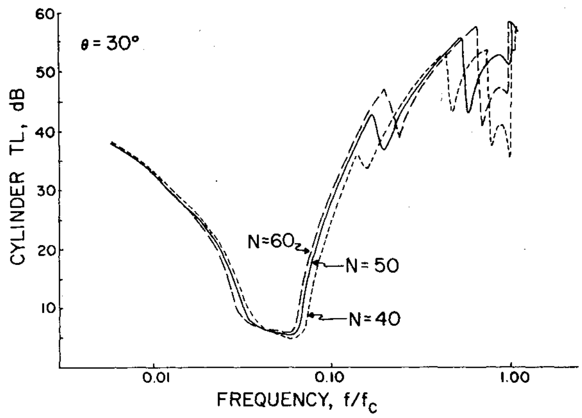

Fig. 5 Variation of TL with the number of stiffeners ($\theta = 30$ deg).

Ref. 1) is presented showing the cylinder transmission loss for a monocoque shell for various incidence angles. Inspection of the figure shows that the transmission loss for a monocoque shell has two major dips, one at the ring frequency $f_R = \omega_R/2\pi = (2\pi a)^{-1} (E/\rho^{sh})^{1/2}$ and the other at the critical frequency $f_c = (c^2/2\pi)(m^{sh}/D)^{1/2}$. Between f_R and f_c, the transmission loss follows a mass-law behavior; below f_R, the TL tends to be stiffness-governed, except where structural resonances† of the cylinder are present. For convenience, the abscissa of Fig. 2 is nondimensionalized relative to the critical frequency f_c. For the dimensions given previously, $f_R = 445$ Hz and $f_c = 7552$ Hz. In Fig. 2, then, the ring-frequency dip occurs at $f_R/f_c = 445/7552 = 0.0589$ and the coincidence frequency at $f/f_c = 1.0$.

Figures 3 and 4 show the effect of stringers on TL for $\theta = 30$ and 45 deg. The number of stringers assumed was 50, uniformly spaced around the circumference. The TL for a monocoque shell (with $\eta = 0.01$) is shown for reference. The dotted curve shows TL for a heavily damped stiffened shell ($\eta = 0.500$). The stringers appear to raise TL in the mass-controlled region, exhibiting dips at stringer rotational resonances. The ring-frequency effect characteristic of monocoque shells is still seen to be present, although high damping seems to help control the size of the dip.

The variation of TL with the number of stiffeners is shown in Figs. 5 and 6 for incidence angles 30 and 45 deg. The curves indicate a small increase in TL as the number of stringers is increased from 40 to 60. This is especially true in the mass-controlled region.

†These cylinder resonances occur when the trace wavelength and frequency of the incident wave match with a shell vibrational mode having the appropriate value of m, the number of circumferential waves.

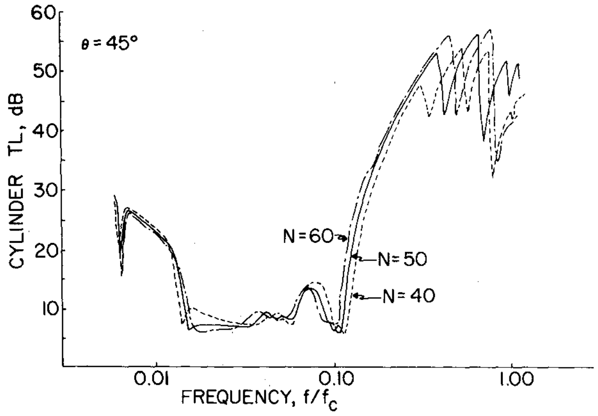

Fig. 6 Variation of TL with the number of stiffeners ($\theta = 45$ deg).

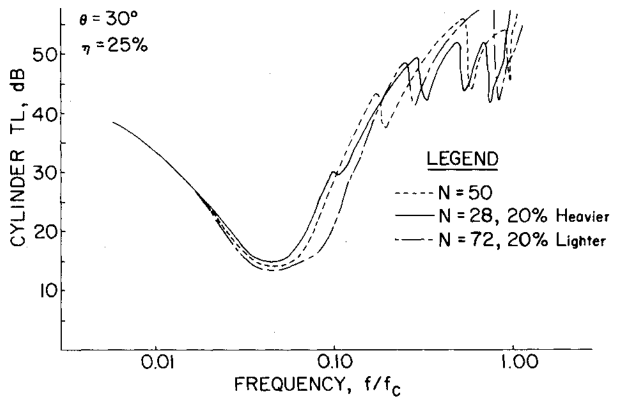

Fig. 7 Effect of stiffener size on cylinder TL ($\theta = 30$ deg).

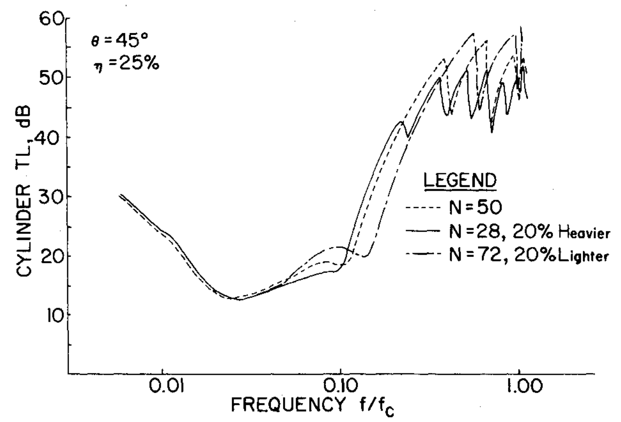

Fig. 8 Effect of stiffener size on cylinder TL ($\theta = 45$ deg).

Figures 7 and 8 show how TL varies with the size of the stringers under the constraint that the total weight of the stringers is maintained unchanged. Thus, with 50 stringers as the reference, the size of the stringer was scaled up (and down) so that the cross-sectional area was increased (decreased) by 20%. To maintain constant total stringer weight, the number of stringers was decreased (increased) by 20%. Inspection of Figs. 7 and 8 fails to reveal a significant trend. Since an airplane fuselage will generally have damping treatment, a loss factor of $\eta = 0.25$ was assumed in Figs. 7 and 8 to try to simulate a realistic situation.

The effect of damping on TL for a stiffened shell is shown in Figs. 9 and 10 for $\theta = 60$ and 75 deg. The number of stringers is 50. For reference, the transmission loss for the monocoque shell (with $\eta = 0.01$) is shown, as is that for a lightly damped stiffened shell. The three solid lines show how TL is affected by higher damping rates, viz., $\eta = 0.25$, 0.50,

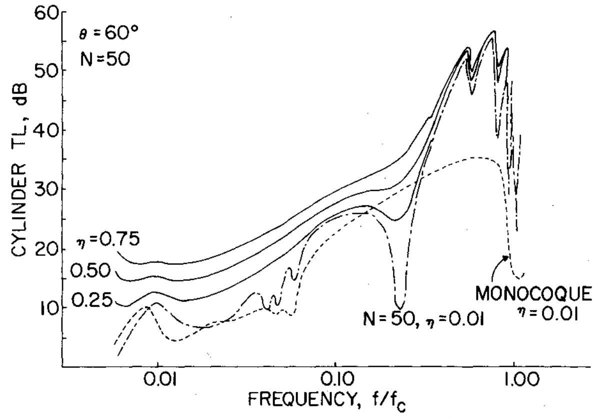

Fig. 9 Cylinder TL for heavily damped stiffened shell ($\theta = 60$ deg).

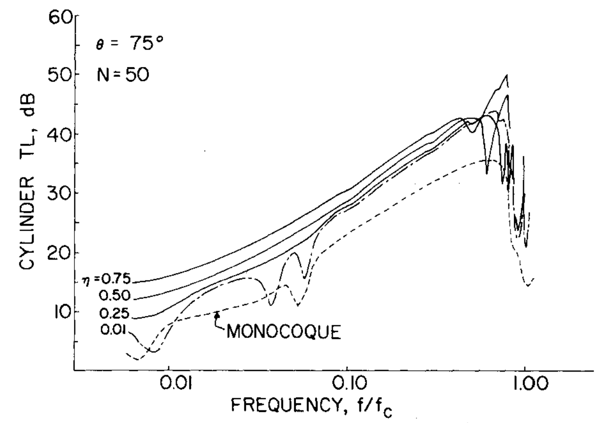

Fig. 10 Cylinder TL for heavily damped stiffened shell ($\theta = 75$ deg).

and 0.75. As is evident, damping smooths out the curves, eliminating dips at resonances and providing modest increases in TL. This is consistent with the results obtained in Ref. 8.

IV. Summary

In the context of the transmission of airborne noise into an aircraft fuselage, a mathematical model is presented for the transmission of an oblique sound wave into a thin stiffened cylindrical shell. The stiffeners are longitudinal stringers and are modeled as discrete structural elements. Numerical cases were examined with numbers typical of narrow-bodied jet transport fuselages.

Stringers appear to raise the cylinder transmission loss in the mass-controlled region, although they produced dips in the TL curve at stringer resonances. The ring-frequency dip in TL which is characteristic of monocoque shells is still present. There is some evidence that stringers may raise the ring frequency by a small amount. There appears to be a small increase in TL as the number of stringers is increased. This was especially true in the mass-controlled region. The effect of high damping was also examined. It is seen to smooth out TL curves, eliminating dips at resonance and providing modest increases in the cylinder transmission loss.

Appendix

Rinehart and Wang[5] have shown that the potential energy of the jth stringer (located at $\phi = \phi_j$) is

$$V_j = \frac{E_s}{2} \int_0^L [A_s (u^*,_x)^2 + \bar{I}_{zz} (v^*,_{xx})^2 + \bar{I}_{yy} (w^*,_{xx})^2 + 2\bar{I}_{yz} (v^*,_{xx}) (w^*,_{xx}) + C_w (\theta,_{xx})^2 + GJ(\theta,_x)^2]_j \, dx \tag{A1}$$

where u^*, v^*, w^* are the displacement components of the shear center, θ denotes the angle of twist, C_w is the warping rigidity, E_s is the Young's modulus for the stringer, GJ is the torsional rigidity, A_s is the stringer area, the subscript j denotes the jth stringer, and $\bar{I}_{yy}$, $\bar{I}_{zz}$, $\bar{I}_{yz}$ are area moments and products of inertia about centroidal axes. Commas followed by coordinates in the subscripts denote derivatives, e.g., $u^*,_x = \partial u^*/\partial x$, etc. If u, v, w are shell displacement components, then the stringer shear center displacements are given by

$$\begin{aligned} u^* &= u + y_s\, v,_x + z_s w,_x \\ v^* &= v + z_s\theta \\ w^* &= w - y_s\theta \\ \theta &= w,_y \end{aligned} \tag{A2}$$

where (y_s, z_s) represents the location of the attachment line with respect to the shear center.

If u', v', w' are the displacement components of an element in the stringer, and u, v, w are shell displacement components, then the velocity components of an infinitesmal element in the stringer are

$$\begin{aligned} \dot{u}' &= \dot{u} - z\,\dot{w},_x - y\,\dot{v},_x \\ \dot{v}' &= \dot{v} - z\,\dot{w},_y \\ \dot{w}' &= \dot{w} + y\,\dot{w},_y \end{aligned} \tag{A3}$$

where (y,z) are the coordinates of the element relative to the center of attachment. The kinetic energy of the stringer at $\phi = \phi_j$ is then (integration is over the cross-sectional area of the stringer)

$$T_j = \frac{\rho_j}{2} \int_0^L \int_A \int (\dot{u}'^2 + \dot{v}'^2 + \dot{w}'^2)\,\mathrm{d}A\,\mathrm{d}x \tag{A4}$$

Introduction of (y_c, z_c), the coordinates of the centroid of the stringer cross-sectional area, via

$$y = y_c + \hat{y} \qquad z = z_c + \hat{z} \tag{A5}$$

and substitution of Eqs. (A3) and (A5) into Eq. (A4) leads to

$$\begin{aligned} T_j = \frac{\rho_j}{2} \int_x \{ & A_s [(\dot{u} - z_c\,\dot{w},_x - y_c\dot{v},_x)^2 + (\dot{v} - z_c\,\dot{w},_y)^2 \\ & + (\dot{w} + y_c\,\dot{w},_y)^2] + \bar{I}_{yy} (\dot{w},_x)^2 + \bar{I}_{zz} (\dot{v},_x)^2 \\ & + \bar{I}_p (\dot{w},_y)^2 + 2\bar{I}_{yz} (\dot{w},_x)(\dot{v},_x) \}_j \mathrm{d}x \end{aligned} \tag{A6}$$

where ρ_j is the material density of the stringer, and

$$\bar{I}_p = \bar{I}_{yy} + \bar{I}_{zz}$$

In addition to the strain and kinetic energies V_j and T_j, there are the standard expressions for the strain and kinetic energies of the cylindrical shell, viz.,

$$\begin{aligned} V_s = \frac{1}{2} \frac{Eha}{(1-\nu^2)} \int_0^L \int_0^{2\pi} & [(e_{xx} + e_{yy})^2 - 2(2-\upsilon) \\ & \times (e_{xx}e_{yy} - e_{xy}{}^2)]\,\mathrm{d}x\mathrm{d}\phi \\ + \frac{1}{2} Da \int_0^L \int_0^{2\pi} & [(K_{xx} + K_{yy})^2 - 2(1-\nu) \\ & \times (K_{xx}K_{yy} - K_{xy}{}^2)]\,\mathrm{d}x\mathrm{d}\phi \end{aligned} \tag{A7}$$

$$T_s = \frac{m^{sh} a}{2} \int_0^L \int_0^{2\pi} (\dot{u}^2 + \dot{v}^2 + \dot{w}^2)\,\mathrm{d}x\mathrm{d}\phi \tag{A8}$$

where $D = Eh^3/12(1-\nu^2)$, m^{sh} is the shell mass per unit area, h is the shell wall thickness, and

$$\begin{aligned} e_{xx} &= u,_x & e_{yy} &= v,_y + w/a \\ e_{xy} &= \tfrac{1}{2}(u,_y + v,_y) \\ K_{xx} &= w,_{xy} & K_{yy} &= w,_{yy} \\ K_{xy} &= w,_{xy} + (u,_y - v,_x)/2a^2 \end{aligned} \tag{A9}$$

The total kinetic and strain energies are then

$$T = T_s + \sum_{j=1}^{N} T_j \qquad V = V_s + \sum_{j=1}^{N} V_j \tag{A10}$$

where N is the number of stringers.

Use of Eq. (A10) in Hamilton's principle and use of the standard procedures from the calculus of variations generate the equations

$$\begin{aligned} & L_{11}(u) + L_{12}(v) + L_{13}(w) + \alpha_s \sum_{j=1}^{N} (u,_{xx} + Z_s w,_{xxx} + Y_s v,_{xxx})_j \\ & = \gamma\Big\{ \ddot{u} + \mu_s \sum_{j=1}^{N} (\ddot{u} - Z_c\ddot{w},_x - Y_c\ddot{v},_{xx})_j \Big\} \end{aligned} \tag{A11}$$

$$\begin{aligned} & L_{21}(u) + L_{22}(v) + L_{23}(w) \\ & - \alpha_s \sum_{j=1}^{N} (Y_s u,_{xxx} + I'_{zz}\, v,_{xxx} + I'_{yz}\, w,_{xxxx} + I'''_{zz}\, w,_{xxxx\phi})_j \\ & = \gamma\Big\{ \ddot{v} + \mu_s \sum_{j=1}^{N} (\ddot{v} - Z_c\ddot{w},_\phi + I''_{zz}\,\ddot{v},_{xx} + I''_{yz}\,\ddot{w},_{xx} - Y_c\,\ddot{u},_x)_j \Big\} \end{aligned} \tag{A12}$$

$$\begin{aligned} & L_{31}(u) + L_{32}(v) + L_{33}(w) \\ & + \alpha_s \sum_{j=1}^{N} (Z_s u,_{xxx} + I'_{yy} w,_{xxxx} + GJ' w,_{xx\phi\phi} \\ & + I'_{yz} v,_{xxxx} - I'''_{zz} v,_{xxxx\phi} - C'_w\, w,_{xxxx\phi\phi})_j \\ & = -\gamma\Big\{ \ddot{w} + \mu_s \sum_{j=1}^{N} (\ddot{w} - Z_c\ddot{u},_x + Z_c\ddot{v},_\phi - I'_y{}'\,\ddot{w},_{xx} \\ & - I''_{yz}\,\ddot{v},_{xx} - I'_p\,\ddot{w},_{\phi\phi})_j \Big\} + p \end{aligned} \tag{A13}$$

where L_{ij} are the familiar operators in Flügges equations for a monocoque shell,[9] p is the exciting pressure, and

$$\alpha_s = \frac{E_s A_s (1-\nu^2)}{Eha} = \text{stringer stiffness parameter} \tag{A14}$$

$$\mu_s = \frac{\rho_s A_s}{m^{sh} a} = \text{stringer mass parameter} \tag{A15}$$

$$I'_{yy} = Z_s^2 + I_{yy}/A_s a^2 \tag{A16}$$

$$I'_{zz} = Y_s^2 + I_{zz}/A_s a^2 \tag{A17}$$

$$I'_{yz} = Y_s Z_s + I_{yz}/A_s a^2 \tag{A18}$$

$$I''_{zz} = Y_c^2 + I_{zz}/A_s a^2 \tag{A19}$$

$$I''_{yy} = Z_c^2 + I_{zz}/A_s a^2 \tag{A20}$$

$$I''_{yz} = Y_c Z_c + I_{yz}/A_s a^2 \tag{A21}$$

$$I'''_{zz} = (Z_s I_{zz} - Y_s I_{yz})/A_s a^2 \tag{A22}$$

$$I'_p = Y_c^2 + Z_c^2 + I_p/A_s a^2 \tag{A23}$$

$$GJ' = GJ/E_s A_s a^2 \tag{A24}$$

$$C'_w = (C_w/a^2 + Z_s^2 I_{zz} + Y_s^2 I_{yy})/A_s a^2 \tag{A25}$$

$$Z_s = z_s/a \qquad Y_s = y_s/a \tag{A26}$$

$$Z_c = z_c/a \qquad Y_c = y_c/a \tag{A27}$$

$$\gamma = m^{sh}(1-\nu^2)a^2/Eh \tag{A28}$$

In Eqs. (A14) and (A15), α_s and μ_s are consistent with similar parameters obtained by Egle and Sewall.[10]

For the pressure pulse,

$$P = P_0 \exp[j(\omega t - x_{mz}z]\cos m\phi \tag{A29}$$

Displacements can be taken as

$$\begin{Bmatrix} u \\ v \\ w \end{Bmatrix} = \sum_{m=1}^{\infty} \begin{Bmatrix} jU_m & \sin m\phi \\ V_m & \cos m\phi \\ W_m & \cos m\phi \end{Bmatrix} \exp[j(\omega t - x_{mz}z)] \tag{A30}$$

and the modal impedance developed in the form

$$Z_m^{sh} = j\, m^{sh}\, \omega \frac{(\omega_R/\omega)^2 \Delta}{(1-\nu^2)[A_{21}A_{12} - A_{11}A_{22}]} \tag{A31}$$

where

$$\Delta = \det[A_{ij}] \tag{A32}$$

$$A_{11} = (1 + \tfrac{1}{2}P' + \alpha_s \sigma_{cc})x_{mz}^2 + \{[(1-\nu)/2](1+K) - P'\}m^2 - \Omega(1+\mu_s\sigma_{cc}) \tag{A33}$$

$$A_{12} = A_{21} = [(1+\nu)/2]m x_{mz} - \sigma_{sc}(\alpha_s Y_s x_{mz}^3 + \Omega\mu_s Y_c x_{mz}) \tag{A34}$$

$$A_{13} = A_{31} = (\nu + P')x_{mz} + K x_{mz}\{x_{mz}^2 - [(1-\nu)/2]m^2\} - \sigma_{cc}(\alpha_s Z_s x_{mz}^3 + \Omega\mu_s Y_c x_{mz}) \tag{A35}$$

$$A_{23} = m(1-P') - [(3-\nu)/2]Km\, x_{mz}^2 + \alpha_s(I'_{yz}\sigma_{cc} - I'''_{zz} m\sigma_{ss})x_{mz}^4 - \Omega\mu_s(\sigma_{ss}Z_c\, m + I''_{yz}x_{mz}^2\sigma_{cc}) \tag{A36}$$

$$A_{32} = m(1-P') - [(3-\nu)/2]Km\, x_{mz}^2 + \alpha_s(I'_{yz}\sigma_{sc} - I'''_{zz} m\sigma_{cc})x_{mz}^4 - \Omega\mu_s(\sigma_{cc}Z_c m + I''_{yz}x_{mz}^2\sigma_{cs}) \tag{A37}$$

$$A_{22} = m^2(1-P') + \{[(1-\nu)/2](1+3K) + 0.5P'\}x_{mz}^2 + \alpha_s\sigma_{ss}I'_{zz}x_{mz}^4 - \Omega(1 + \mu_s\sigma_{ss} + I''_{zz}x_{mz}^2\sigma_{ss}) \tag{A38}$$

$$A_{33} = 1 + K[(m^2 + x_{mz}^2)^2 + 1 - 2m^2] + (m^2 + 0.5x_{mz}^2)P' + \alpha_s\sigma_{cc}[I'_{yy}x_{mz}^4 + GJ'm^2x_{mz}^2 + C'_w m^2 x_{mz}^4] - \Omega[1 + \mu_s\sigma_{cc}(1 + I''_{yy}x_{mz}^2 + I'_p m^2)] \tag{A39}$$

where

$$\Omega = \omega^2(1-\nu^2)/\omega_R^2(1+j\eta) \tag{A40}$$

$$P' = p_i(1-\nu^2)a/Eh \tag{A41}$$

$$x_{mz} = k_{1z}a \tag{A42}$$

$$k_{1z} = (\omega/c_1)\cos\theta/(1+M_1\cos\theta) \tag{A43}$$

$$\sigma_{cc} = \sum_{j=1}^{N}\cos^2 m\phi_j \tag{A44}$$

$$\sigma_{ss} = \sum_{j=1}^{N}\sin^2 m\phi_j \tag{A45}$$

$$\sigma_{sc} = \sum_{j=1}^{N}\sin m\phi_j \cos m\phi_j \tag{A46}$$

$$K = h^2/12a^2 \tag{A47}$$

where N is the number (even) of stringers, ϕ_j the location of the jth stringer, and p_i is the internal pressure (to simulate a pressurized cabin).

In formulating Z_m^{sh}, it was assumed that an even number of stringers were uniformly distributed around the circumference so that

$$\phi_j = (2\pi j - \pi)/N \qquad (j = 1, 2, \ldots, N) \tag{A48}$$

Furthermore, Egle and Sewall[10] showed that, when stringers are arranged symmetrically, then symmetric and antisymmetric modes uncouple; and when the number of stringers is large (like $N = 50$), the coupling between modes with different circumferential wave numbers is weak and can be neglected. These simplifications have been adopted in deriving Eq. (A31) for Z_m^{sh}.

Acknowledgments

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